Generation, Propagation, and Dissipation of Internal Waves in Continental Shelf Regimes

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LONG-TERM GOALS:

To understand the dynamics of nonlinear internal waves in littoral regimes, including the processes of generation, propagation, interaction, and dissipation; to formulate analytical models of these processes that have useful predictive skill; and to transition the models to Navy operational interests.

OBJECTIVES

A. Relate generation processes to tidal and meteorological forcing near bathymetric features.

B. Establish the spatial and temporal variability in propagation of solitons on the shelf, as evidenced by the character of the phase fronts of the wave packets, including their refractive control and advection by and interaction with tidal currents.

C. Study soliton-soliton and soliton-current interactions (a) by analyzing intersections of pairs of wave packets for the phase shifts and relating them to theory, and (b) by solving the eigenvalue equation for the vertical structure function in the presence of background flows.

D. Characterize variations in the separation, speed, direction, and number of individual solitons as they propagate up the continental shelf.

E. Establish the area of disappearance of soliton signatures in shallow water; relate this phenomenon to what is known of the stratification, mixing, and turbidity in that region.

F. Use nonlinear soliton theory to derive an analytical model for the waves.

APPROACH

The approach has been to use a combination of in-situ field data and satellite imagery, especially synthetic aperture radar (SAR), together with analytic theories for solitons, to understand the nonlinear dynamics. Participation in the ONR-sponsored 1995 Shallow Water Acoustics in Random Media (SWARM) experiment provided a rich source of quantitative field data on internal waves over the continental shelf off New Jersey. In addition, monitoring soliton results from Gibraltar and from Knight Inlet gives alternative sources of information on generation and propagation near a sill. Also, acquisition of satellite imagery from a variety of locations around the world allows one to glean some overview of the incidence of the phenomenon in general. Such data show that solitons are ubiquitous in stratified coastal oceans and are occasionally seen in deep water; presumably the latter result from formation over shallow topography and propagation out into the open sea.

WORK COMPLETED

A. In cooperation with workers from the Naval Research Laboratory and the Woods Hole Oceanographic Institution, data from SWARM have been reduced and analyzed for internal wave characteristics. Such data include thermistor time-series, ADCP records, CTD and XBT casts, acoustic flow-visualization methods, ship's radar PPI images, and SAR images from the ERS-1 satellite. A major paper has been published (Apel et al., 1997) that includes both acoustic and oceanographic information.

B. An analytic model of soliton packets that was found in the Soviet plasma physics literature has been adapted to the case of a stratified, bounded medium. Because it incorporates the Jacobi elliptic function dn(x, s), it has been termed the "dnoidal" model in analogy to the well-known cnoidal solution to the Korteweg-De Vries equation, cn(x, s). The model reproduces the major characteristics of solitons observed in nature, including finite packet length, continual lengthening of the packets and number of solitons in them, varying amplitudes and wavelengths within the groups, and a long-term depression of the thermocline in the wake of the packets. Additional modifications to the model, as described below, allow it to mimic entire semidiurnal cycles of internal soliton/tide activity as sequences of undulatory bores, or "solibores" (Henyey et al., 1995).

C. A model for the vertical structure function giving the variation of soliton amplitude with depth has also been formulated in terms of a modified Taylor-Goldstein equation having a dissipative right-hand-side similar to the Orr-Sommerfeld equation. The existence of unstable solutions has been demonstrated for shear flows having critical levels; the criteria for such instability has been shown to meet the conditions for "over-reflection" as given by Lindzen and associates (Lindzen, 1988).

D. Analysis of SAR images for soliton properties has been carried out for several images and the resultant profiles compared with field data.

RESULTS

The soliton model is in part an adaptation to internal waves of the little-known solution of Gurevich and Pitaevskii (1973), originally derived for collisionless shock waves in plasma. The Jacobi elliptic solution for the amplitude, $\eta(r, z, t)$ is modified by multiplying it by (1) a second function $I(r, t)$ describing the highly nonlinear internal tidal bore and (2) by the vertical structure function, $W_k(z)$, which is obtained from solutions to the modified Taylor-Goldstein equation mentioned above. The complete two-dimensional solution is thus written

$$\eta(r,z,t) = \sum_{n=1}^{N} \eta_{o,n} W_{k,n}(z)\ \exp\left(-\alpha_n r\right) \left\{ 2\,\mathrm{dn}_s^2\left[\tfrac{1}{2} k_o\left(\mathbf{n}\cdot\mathbf{r} - V_n(s)\,t\right)\right] - \left(1 - s^2\right)\right\} I(r,t)$$

where the summation is over the vertical eigenmodes. Attenuation is incorporated via the exponential factor. The nonlinear parameter s is given by Gurevich and Pitaevskii in terms of an implicit function that describes its variation throughout the length of a wave packet: $0 \le s \le 1$, with full nonlinearity occurring at the front of the group and decaying to linear behavior at the rear. This variation directly gives the variable wavelengths of the individual solitons within the packet via an expression for the wavelength, $\lambda = 4\,K(s)/k_0$, where k_0 is the linear wave number at the rear of the packet (also related to the width of each individual soliton). However, any other reasonable specification of the nonlinear parameter may be used, including ones derived experimentally. Thus the model can be tuned to actual data, if desired, by observing the wavelengths within a packet.

An example of three cycles of soliton packets, each separated by one semidiurnal period, is illustrated in Fig. 1 for conditions extant during the SWARM period. The waveforms are shown as displacements of a constant density surface; in this case, the recovery of the pycnocline following the passage of the solitons is given by the relation $I(r, t) = -\{1 + \tanh[(r - c_0\, t)/L]\}$, where c_0 is the long-wave phase speed and L is a horizontal scale. Parameters have been somewhat tuned to reproduce observed packets.

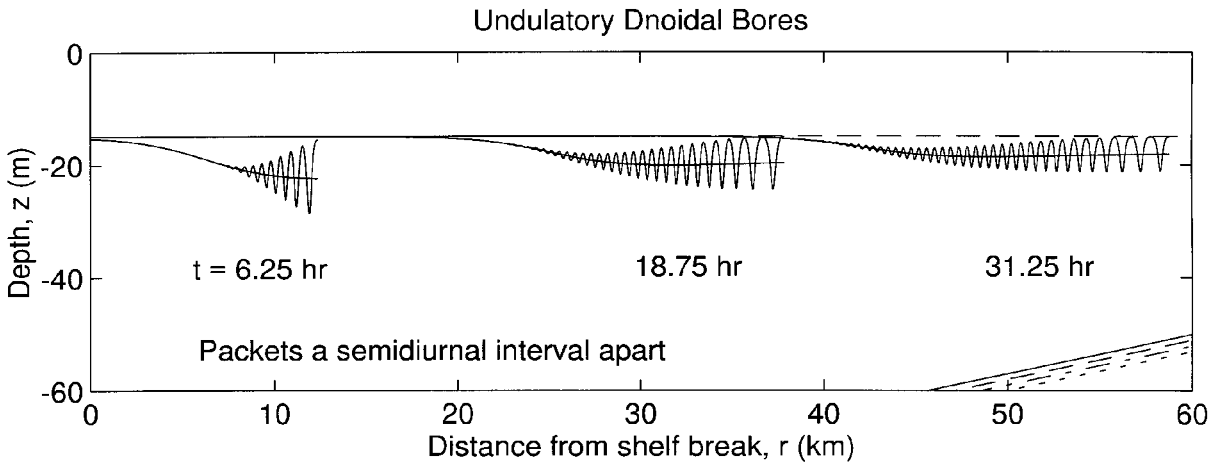

Figure 2. Dnoidal wave packets propagating up the continental shelf after formation at the shelf break.

The relationship governing the vertical structure function is a combination of the Taylor-Goldstein and Orr-Sommerfeld equations. While the usual soliton solution assumes infinite wavelength to be the case, the finite lengths observed in nature and their decay to short-wavelength linear states at the rear of the packet require the use of a more complicated version with $k \neq 0$. In addition, the possibility of shear-flow instability for slow internal waves moving in background currents necessitates the inclusion of such flows in the formulation. The presence of damping mechanisms such as eddy viscosity brings in a lossy right-hand side. The equation to be used is thus:

$$\frac{d^2 W_k}{dz^2} + k^2\left(1 - \frac{N^2(z)}{\left[\omega - \mathbf{k}\cdot\mathbf{U_0}(z)\right]^2} - \frac{d^2 U_0(z)/dz^2}{k\left[\omega - \mathbf{k}\cdot\mathbf{U_0}(z)\right]}\right) W_k =$$

$$\left(\frac{iA}{\omega - \mathbf{k}\cdot\mathbf{U_0}}\right)\left\{W_k^{\mathrm{iv}} - 2k^2 W_k^{\mathrm{ii}} + k^4 W_k\right\}$$

Away from critical layers in the fluid ($\omega/k = U_0$), the right-hand side has the simple effect of making the frequency ω complex with a wave-number-dependent loss term: $\omega = \omega_r + i\omega_i(k)$. Growing waves must thus overcome whatever dissipation (represented by eddy viscosity *A)* before becoming unstable.

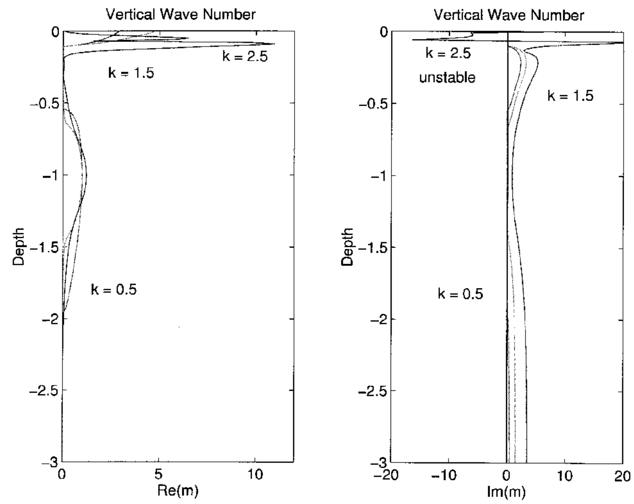

Figure 5. Real and imaginary parts of vertical wave number. Unstable growth is indicated for $k = 2.5$

On the left-hand side one can consider the quantity in the curly braces to be the squared vertical component, m^2 of the wave vector, which itself varies in z. By inspecting the sign of the imaginary part of m, the presence or absence of instability can be discerned without actually solving the differential equation for complex eigenfunctions and eigenvalues. By using models for $N(z)$ and $U_0(z)$ and a model dispersion relation, $\omega = \omega(k)$, the vertical dependence of m can be investigated for a variety of buoyancy and flow profiles. This has been done and the results of Lindzen et al. (1988) were verified: Unstable conditions arise when upward-propagating internal waves tunnel through an evanescent region into a propagating region wherein simultaneously the Richardson's number $R_i \leq 1/4$. For nominal oceanic shear flows, it is slow, short internal waves that are first affected. Figure 2 shows the vertical wave number dependence for unstable conditions wherein the phase speed of the waves matches the current speed near the surface, resulting in a complex wave number component with a negative imaginary part (Fig. 2, right). In the region of $m_i \leq 0$, $R_i \leq 0.25$ as well and instability results.

A SAR image of a large soliton packet being radiated from the Strait of Gibraltar is shown in Fig.3. (The image is in corrected geographical coordinates.) It is known (Apel et al., 1988) that variations in relative radar backscatter intensity such as observed in the radar image are related to the product of variations in the horizontal strain rates of the soliton currents at the surface and the short-surface-wave relaxation time, τ, viz: $\Delta\sigma^o/\sigma^o \approx 4.5\,(\partial u/\partial x)_0\,\tau$. For 2-dimensional flows, the continuity equation and the linearized kinematic boundary condition combined with the cross section data give

$$\frac{\Delta\sigma^o}{4.5\,\sigma^o\,\tau} = \left(\frac{\partial u}{\partial x}\right)_0 = -\left(\frac{\partial w}{\partial z}\right)_0 \simeq -\omega\,\eta\left(\frac{dW_k}{dz}\right)_0$$

Thus if one calculates the vertical structure function $W_k\,(z)$ from CTD data and measures the wavelength and its variations from image data, it is possible to estimate the amplitude and the entire subsurface hydrodynamics using this simple theory. From preliminary analysis, the method appears to give amplitudes to well within a factor of two. Further work will be done on this problem.

IMPACT/APPLICATIONS

This work finds immediate application to ocean acoustics in the littoral zone. From SWARM results, it appears that solitons on the continental shelf can lead to acoustic intensity fluctuations of order ± 10 dB over times of the order of internal wave periods. By combining satellite or aircraft imagery with historical CTD/STD/XBT data, it seems possible to estimate the level of solitary internal wave fluctuations in other regions of the world without the direct physical probing that is usually required. Another application is to nutrient and larval transport; large directional flows and bottom interactions accompany the solitons and these have impacts on the biological state of affairs where they occur.

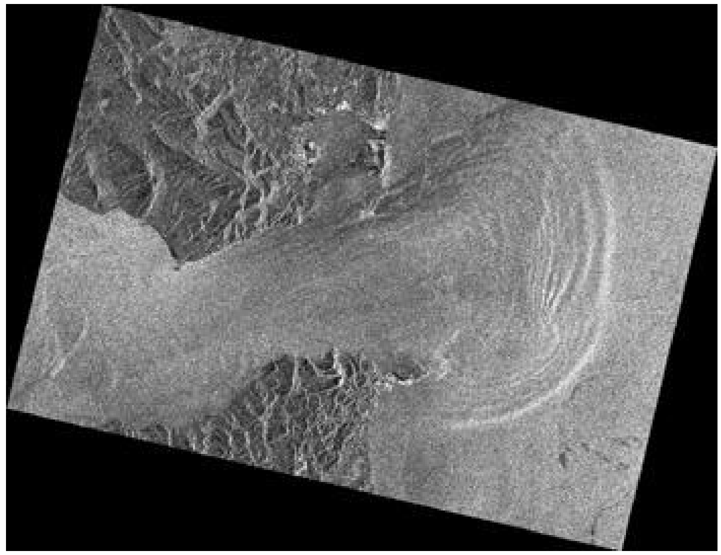

Figure 6. Large soliton packet radiating from Gibraltar in April 1996. Data from SAR on ERS-1.

TRANSITIONS

Acoustic modelers in Navy laboratories and academia are using the model to calculate sound propagation through the internal wave field; usually that field is taken as a combination of deterministic solitons and a random collection of linear waves characterized by modified Garrett-Munk-like spectra.

RELATED PROJECTS

This project is closely related to the ONR-sponsored programs in Mediterranean outflow through the Gibraltar Strait (P. Worcester et al.) and the solibore project in Knight Inlet, Canada (D. Farmer et al.). It is also of relevance to ongoing shallow water acoustics studies in the U.S. and abroad.

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